WS 7

Average Rates of Change

1) $f(x) = \begin{cases} x^2+x+a & x<1 \\ x^3 & x \geq 1 \end{cases}$

$\lim_{x\to 1^-}(x^2+x+a) = 2+a$

$\lim_{x\to 1^+} x^3 = 1$

$2+a = 1$

$\boxed{a = -1}$

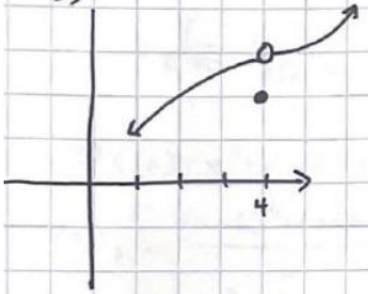
2) $f(x) = e^{-x} - x = 0$ $f(x)$ is cont.

$f(0) = 1 > 0$

$f(1) = \frac{1}{e} - 1 < 0$

$e^{-x} = x$ has a solution in $(0,1)$

3)



4) $f(x) = x^3 + 1$

a) $ROC = \frac{f(3)-f(2)}{3-2}$

$= 28 - 9$

$= 19$

b) $\frac{f(1)-f(-1)}{2} = 1$

5) $f(x) = e^x$

a) $ROC = \frac{f(0)-f(-2)}{2}$

$= \frac{e - \frac{1}{e^2}}{2}$

b) $ROC = \frac{f(3)-f(1)}{2}$

$= \frac{e^3 - e}{2}$

6) $f(x) = \ln x$

a) $ROC = \frac{f(4)-f(1)}{3}$

$= \frac{\ln 4}{3}$

b) $ROC = \frac{f(103)-f(100)}{3}$

$= \frac{\ln 103 - \ln 100}{3}$

$= \frac{1}{3}\ln\frac{103}{100}$

7) $f(x) = 2 + \cos x$

a) $ROC = \frac{f(\pi)-f(0)}{\pi}$

$= \frac{1-3}{\pi} = -\frac{2}{\pi}$

b) $ROC = \frac{f(\pi)-f(-\pi)}{2\pi}$

$= \frac{1-1}{2\pi} = 0$

8) $ROC = \frac{f(2)-f(1)}{1} = 6$

9) $ROC = \frac{f(9)-f(5)}{4} = \frac{44-32}{4} = 3$

10) $m_{x=3} \approx \frac{f(5)-f(2)}{3} = \frac{8}{3}$

11) $Q(t) = 10(4-t)^2$

$$ROC = \frac{Q(2)-Q(0)}{2}$$

$$= \frac{10(4)-10(16)}{2}$$

$$= -60 \text{ gal/min}$$

12) $A(t) = 6.687(0.931)^t$

$$ROC = \frac{A(30)-A(0)}{30}$$

$$= -0.1968 \text{ pounds/day}$$

13) $s(t) = 4.9t^2$

a) $s(2.5) - s(2) = 11.025$ m

b) $ROC = \frac{11.025}{0.5} = 2.205$ m/sec

c)

$$\frac{s(2.01)-s(2)}{0.01} = 19.649 \text{ m/sec}$$

$$\frac{s(2.001)-s(2)}{0.001} = 19.6049 \text{ m/sec}$$

$$\frac{s(2.00001)-s(2)}{0.00001} = 19.600049 \text{ m/sec}$$